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The isentropic relation in plasmas

K T A L Burm

Flemish Institute for Technological Research VITO, Material Technology, Boeretang 200, 2400 Mol, Belgium

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Abstract
The isentropic relation from gas dynamics relates pressure, density and temperature. The use of this relation may ease the hydrodynamic modelling effort. Characteristic for the isentropic relation is the constant isentropic exponent. The isentropic exponent is also in the case of plasmas a constant as long as the ionization degree is between 5 and 80%. This constant is lower due to the extra degree of freedom which comes with ionization in plasmas. The occurrence of ionization means that plasmas are never isentropic. The isentropic relation itself is therefore adapted here to include plasmas within its concept. From the plasma isentropic relation a further extension is to include viscosity and heating. It is found that all extra non-isentropic inclusions further lower the (quasi-) isentropic exponent in the adapted isentropic relation.

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1. Introduction

In gas dynamic theory it is common to use the isentropic relations, which relate pressure, density and temperature to reduce the complexity of the hydrodynamic description. The isentropic relation is valid under the condition that viscosity and heating of the considered gas volume can be neglected. In situations where viscosity and heating play no significant role, the isentropic relations are often used as a first approximation of the complex gas dynamical systems.

The isentropic relations relating the pressure, p, or the temperature, T, and the mass density, ρ, read

$$\frac{p}{\rho^{\gamma}} = C \qquad \frac{T}{\rho^{\gamma-1}} = C' \tag{1}$$

where C (and C') is a constant and γ is the so-called isentropic exponent, which is only a function of how energy is distributed internally in the considered fluid, i.e. of the degrees of freedom. Note that by determining expression (1) the relation for pressure versus temperature is also determined.

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The isentropic exponent, characteristic for the isentropic relations, is a constant in gas dynamic situations. In the case of plasmas, the isentropic exponent may become a function of the ionization degree and deviations from local thermal equilibrium (LTE). Local thermal equilibrium means that the electron temperature does not deviate from the heavy particle (i.e. atoms and ions) temperature (so-called equi-thermal equilibrium) and that ionization does not differ from Saha's ionization–recombination equilibrium. However (as in the case of gases) the isentropic exponent for atomic plasmas is constant as long as the ionization degree is between 5 and 80% [1]. For most atmospheric plasmas which have an electron temperature of about 1 eV, a sufficiently accurate estimation for the isentropic exponent of plasmas is 1.16 [1]. The isentropic exponent constant for plasmas is lower than for gases due to an extra degree of freedom caused by ionization in plasmas.

In this paper the isentropic relation itself (not the characteristic isentropic exponent) is investigated in order to have the same tool to describe plasmas and calorically perfect gases. The plasma considered here consists of atoms, ions and electrons. Plasmas are not in LTE. In order to stress the influence of ionization and the disequilibrium in energy balances, the ionization degree α and the disequilibrium parameter θ for equi-thermal disequilibrium are used. The ionization degree is defined as

$$\alpha = \frac{n_i}{n_h} \tag{2}$$

in which n_h, the heavy particles density, is equal to the sum of the atom density, n_a, and the ion density, n_i. The disequilibrium parameter θ is defined as

$$\theta = \frac{T}{T_e} \tag{3}$$

in which T is the heavy particle temperature and in which T_e is the electron temperature.

Further, we will consider only singly ionized ions and make use of quasi-neutrality, i.e. the ion density equals the electron density $n_i = n_e$.

2. Plasma thermodynamics

In this section we will consider plasmas along a streamline neglecting viscosity and heating. In [1], the heat capacities (at constant pressure c_p and at constant volume c_V) and the isentropic exponent of a plasma γ_p are considered. From [1] it can be derived that

$$\beta = \frac{c_p}{c_V} = \gamma_p \left[1 + \frac{\alpha(1-\alpha)}{2\theta + \alpha(1-\theta)}\right] \tag{4}$$

and

$$c_p - c_V = (\beta - 1)c_V = R\left(\frac{\theta+\alpha}{\theta}\right) \times \left(1 + \frac{(\beta-\gamma_p)}{\beta\gamma_p}\left[\beta\left(\frac{5}{2} + \frac{E^{\text{ion}}}{kT_e}\right)^2 - \gamma_p\left(\frac{3}{2} + \frac{E^{\text{ion}}}{kT_e}\right)^2\right]\right). \tag{5}$$

Note that in the case of a gas $\alpha = 0$ and $\theta = 1$. This yields

$$\beta = \frac{c_p}{c_V} = \gamma_p = \gamma \tag{6}$$

and

$$c_p - c_V = (\gamma - 1)c_V = R. \tag{7}$$

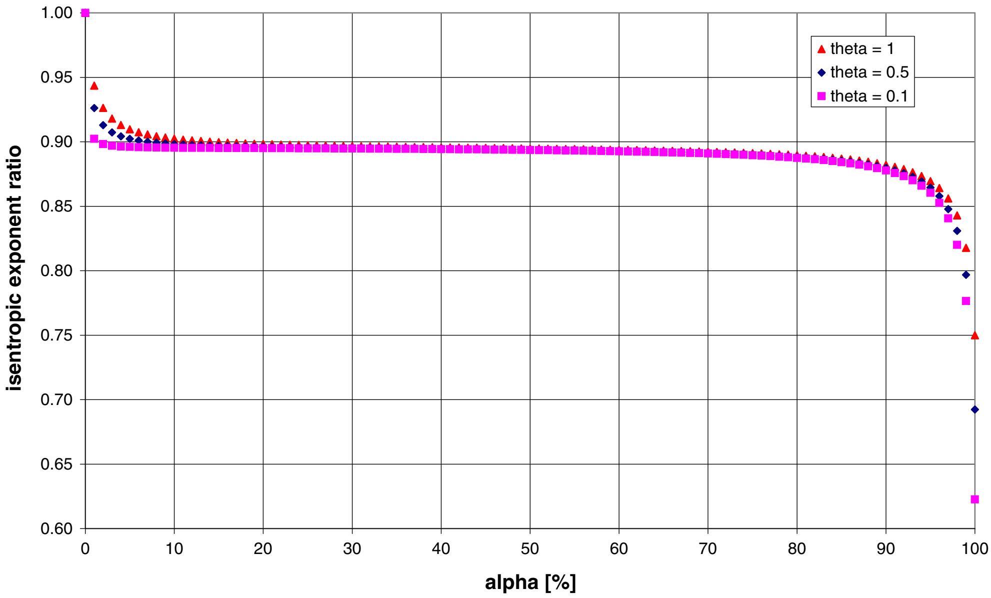

Figure 1. The isentropic exponent ratio (the ratio of γ^* and γ_p) of argon plasmas with an electron temperature of 1 eV. Theta is the temperature disequilibrium parameter defined in expression (3).

Let us define

$$\delta = \frac{c_p - c_V}{R} \tag{8}$$

such that $\delta = 1$ in the case of a gas.

The isentropic continuity equations written in differential form [2] can be combined to

$$\mathrm{d}p = \rho c_p \, \mathrm{d}T. \tag{9}$$

While making use of β and δ as given in expressions (4) and (8), expression (9) can be written as

$$\frac{(\beta - 1)}{\delta} c_V \left(\frac{\mathrm{d}\rho}{\rho} + \frac{\mathrm{d}T}{T} \right) = \beta c_V \frac{\mathrm{d}T}{T}. \tag{10}$$

This yields for expression (9) adapted for a plasma that

$$p = C\rho^{\gamma^*} \tag{11}$$

where C is a constant and γ^* is defined as

$$\gamma^* = \left(\frac{\delta\beta}{(\delta - 1)\beta + 1} \right). \tag{12}$$

In the case of a gas $\gamma^* = \gamma$. Note that expression (11) is not the isentropic relation as known from gas dynamics, i.e. expression (1) with γ replaced by the isentropic exponent for plasmas γ_p.

Figure 1 shows the isentropic exponent ratio, i.e. the ratio of the plasma isentropic relation coefficient (γ^*) and the isentropic exponent for plasmas (γ_p taken from [1]), for argon plasmas at $T_e \approx 1$ eV as a function of the ionization degree for several values of the disequilibrium parameter θ. From figure 1 we conclude that for the ionization range 5–80% a good estimate for the isentropic exponent ratio is 0.9, and that this ratio is roughly independent of θ. The isentropic exponent ratio depends only weakly on the plasma composition and the electron temperature with respect to its behaviour as function of the ionization degree and the disequilibrium parameter θ. The constant (the value 0.9 for argon plasmas at T_e is 1 eV) depends on the ratio of E^{ion} and kT_e.

In the limit that the electron temperature goes to zero, the isentropic exponent γ^* goes to the value of the isentropic exponent for plasmas γ_p. The isentropic exponent γ^* is always less than the isentropic exponent for plasmas γ_p.

Since expression (11) is derived from the expressions of conservation of mass, momentum and energy, while neglecting viscosity and heating, we will call expression (11) the plasma isentropic relation.

A very interesting consequence of expressions (10) and (11) is that

$$c_p = \frac{\beta\delta}{\beta - 1} R = \frac{\gamma^*}{\gamma^* - 1} R \tag{13}$$

something we will use in the next section.

3. General thermodynamics

Let us next consider the flow behaviour of a fluid flowing through a duct with different cross-sections $A = \pi r^2$ at the inlet and the outlet. Mass conservation in differential form gives

$$\mathrm{d}\rho u A = 0. \tag{14}$$

The momentum balance in first order yields [3]

$$\mathrm{d}p + \rho u \, \mathrm{d}u + \frac{\rho u^2}{2} f \frac{2}{r} \, \mathrm{d}z = 0 \tag{15}$$

where z is the symmetry axis coordinate and f the dimensionless Fanning friction factor [4–8]. The Fanning friction factor is a function of the Reynolds number, Re

$$Re = \frac{2\rho u r}{\mu} \tag{16}$$

where μ is the dynamic viscosity. For laminar gas flows ($Re < 2300$) the dimensionless Fanning friction factor is equal to [4–8]

$$f = \frac{16}{Re}. \tag{17}$$

The Fanning friction factor expression is valid for any wall roughness since the heat loss in laminar flow is independent of wall roughness [9].

Conservation of energy yields

$$c_p \, \mathrm{d}T + u \, \mathrm{d}u = \mathrm{d}Q, \tag{18}$$

where $\mathrm{d}Q$ is the rate of heating and T the (gas mixture or plasma) temperature.

Expressions (14), (15) and (18) consider the particles in the fluid as grouped in a volume. The expressions take the total fluid into account integrated over the cross-section and no radial dependences are assumed.

Combining expressions (15) and (18) a similar expression to (9) is found

$$\mathrm{d}p + \rho u^2 \frac{f}{r} \, \mathrm{d}z + \rho \, \mathrm{d}Q = \rho c_p \, \mathrm{d}T. \tag{19}$$

Similar to before, obtaining expression (10) from expression (9), expression (19) can be written as

$$\left(\frac{\mathrm{d}\rho}{\rho} + \frac{\mathrm{d}T}{T} \right) + \frac{u^2}{RT} \frac{f}{r} \, \mathrm{d}z + \frac{\mathrm{d}Q}{RT} = \frac{\gamma^*}{\gamma^* - 1} \frac{\mathrm{d}T}{T} \tag{20}$$

where expression (13) is used. Note that by defining the quasi-heat capacity S and heat flux Γ,

$$S = -\frac{\mathrm{d}Q}{\mathrm{d}T} \tag{21}$$

and

$$\Gamma = -\kappa \frac{\mathrm{d}T}{\mathrm{d}z} \tag{22}$$

with κ the thermal heat conductivity coefficient, expression (20) becomes

$$\frac{\mathrm{d}\rho}{\rho} = \left(\frac{1}{\gamma^* - 1} + \frac{u^2}{R} \frac{\kappa f}{r\Gamma} + \frac{S}{R} \right) \frac{\mathrm{d}T}{T}. \tag{23}$$

This yields the relation

$$p = C\rho^{\gamma^{**}} \tag{24}$$

with a new exponent

$$\gamma^{**} = 1 + \left(\frac{1}{\gamma^* - 1} + \frac{u^2}{R} \frac{\kappa f}{r\Gamma} + \frac{S}{R} \right)^{-1}. \tag{25}$$

Heating and viscosity lower the (isentropic) exponent γ^{**} for plasmas as well as for gases, since heating and viscosity introduce extra degrees of freedom. When viscosity and heating are zero $\gamma^{**} = \gamma^*$ (note the special case of a gas, no viscosity and adiabatic: $\gamma^{**} = \gamma^* = \gamma$).

The viscosity term in expression (25) is roughly constant since at higher temperatures, an increase in temperature results in a strong increase in velocity u as well as a severe decrease in friction factor f (or in viscosity coefficient μ) [11, 12] since the ion contribution in the viscosity term becomes dominant. At relative low temperatures an increase in temperature increases the velocity as well as the viscosity coefficient, but these increases are less severe. At extreme low temperatures the viscosity term can be neglected [11]. The heat capacity term is also roughly constant as long as a change in temperature goes linear with a change in heat addition.

As a consequence of expressions (23) and (24) the quasi-heat capacity c_p^* is given as

$$c_p^* = \frac{\gamma^{**}}{\gamma^{**} - 1} R = \frac{\gamma^*}{\gamma^* - 1} R + u^2 \frac{\kappa f}{r\Gamma} + S. \tag{26}$$

4. Consideration

In gas dynamic theory it is common to use the isentropic relations, which relate pressure, density and temperature to reduce the complexity of the hydrodynamic description. The isentropic relation is valid under the condition that viscosity and heating of the considered gas volume can be neglected. The isentropic exponent, characteristic for the isentropic relations, is a constant in gas dynamic situations, and as long as the ionization degree is between 5 and 80% for atomic plasmas [1]. The isentropic exponent constant for plasmas is lower than for gases due to an extra degree of freedom caused by ionization in plasmas.

However, since plasmas are always non-isentropic due to the occurrence of ionization, the isentropic relation as used for gases (or gas mixtures) has been considered. The plasma isentropic relation has been introduced as an adapted version of the isentropic relation from gas dynamics. In this adapted version, viscosity and heating are still neglected but the relation is not fully isentropic allowing for the inclusion of plasmas in the relation. When viscosity and heating are also included the adapted plasma isentropic relation is found, which has a further lowered isentropic exponent.

To show the usefulness of the plasma isentropic relations we consider the cascaded arc of [10]. This atmospheric argon plasma source is in disequilibrium $\theta \approx 0.8$–0.9 with a typical electron temperature of 1 eV. The ionization degree of a cascaded arc with a straight plasma channel is typically between 6 and 10%. Without estimating the quasi-heat capacity and the heat flux, S and Γ, from experimental values, it can be deduced from expression (25) that

$$1 \leqslant \gamma^{**} \leqslant \gamma^{*}.$$

With the help of figure 1, $\gamma^{*} \approx 0.9 \times 1.16 = 1.044$ ($\gamma_p = 1.16$ is taken from [1]) for the ionization range 5–80%. Noting that the pressure over the cascaded arc drops typically a few 10^4 Pa (depending on the length of the arc), the plasma isentropic relation informs us that the mass density ρ drops one order in magnitude over the cascaded arc while the temperature over the arc does not change (much) at all. This is indeed in agreement with our experience.

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